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WE-Rec: A fairness-aware reciprocal recommendation based on Walrasian equilibrium☆

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ABSTRACT

The emergence of online dating and recruiting platforms brings big challenges to the reciprocal recommendation which has attracted a lot of research attention. Most previous approaches improved the accuracy and diversity of reciprocal recommendations, but few researcher made efforts on the fairness-aware recommendation which aims to avoid the discrimination and mistreatment of vulnerable groups. In this paper, we concentrate on the research of fairness-aware recommendations in the reciprocal recommender system and propose an approach to rerank the recommendation list by optimizing three significant fairness-aware criteria between parties (i.e., buyers and sellers) based on Walrasian equilibrium: (1) the disparity of service; (2) the similarity of mutual preference; (3) the equilibrium of demand and supply. According to these definitions of fairness, we cast the task of reciprocal recommendation as a multi-objective optimization considering the satisfaction of individuals, the fairness of recommendations, and the market clearing simultaneously. The extensive experiments are conducted on two real-world datasets, and the results demonstrate the effectiveness of our approach.

1. Introduction

In decades, with the rise of Internet and e-commerce, the recommender systems (RSs), such as the books (e.g., Amazon) and the movie recommendations (e.g., MovieLens), have been vigorously developed. Many practical algorithms have been designed, implemented, and deployed to provide satisfactory recommendations instead of entirely depending on the experience of experts. However, in recent years, the emergence of online recruitment (e.g., LinkedIn) and dating (e.g., Jiayuan.com) social networks breaks the mold of typical recommendations where a user will also be recommended to other users [1–4]. This new type of social networks is called the reciprocal recommender system.

The reciprocal RS has attracted a lot of research efforts. However, the progress in the techniques of reciprocal recommendation is still limited. The main reason is that if we cast a reciprocal recommendation task as a typical recommendation problem and ignore the correlations between parties, the techniques in typical RSs seem to work in the reciprocal RS [5,6]. In fact, there does exist a gap between the typical and reciprocal RS. Fig. 1(a) is an example of typical item recommendation. In this scenario, consumers can freely choose items where solid lines represent the preference of consumers. Fig. 1(b) illustrates an example of reciprocal recruiting recommendation where dashed and solid lines represent the preference of applicants and companies, respectively. In traditional RSs (i.e., Fig. 1(a)), we only concern about the preferences of consumers who can buy everything if they like it. The successful transaction is based on the one-way selection of consumers because the sellers do not select buyers and the goal of sellers is to sell products for the profit as much as possible. However, In reciprocal RSs (i.e., Fig. 1(b)), it is necessary to simultaneously concern about the preferences of buyers (e.g., companies) and sellers (e.g., applicants) because a buyer and a seller who mutually select each other comprise an ideal situation. In addition, such as the online recruitment recommender system, the published jobs of a company are limited and the occupation is unique for each applicant generally. Therefore, the demand of user (e.g., for both buyers and sellers) in reciprocal RSs is restrictive. The comparison between traditional and reciprocal RSs are summarized in Table 1.

Besides the effectiveness of recommendation, the fairness of recommendation also plays a crucial role in the reciprocal RS. For example, in the news RS (i.e., the traditional RS), the ideal situation is that all users read the news which meet their preferences.

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Table 1
The comparison of characteristics between traditional and reciprocal recommender systems.

Characteristics	Recommender system	
	Traditional	Reciprocal
Target Users	Only **Buyers** (e.g., customer and consumer)	**Buyers** (e.g, company) & **Sellers** (e.g., applicant)
User demands	**Unlimited** for buyers	**Limited** for both buyers and sellers
Successful recommendation	According to the **one-way** selection of buyers	According to the **mutual selections** between buyers and sellers

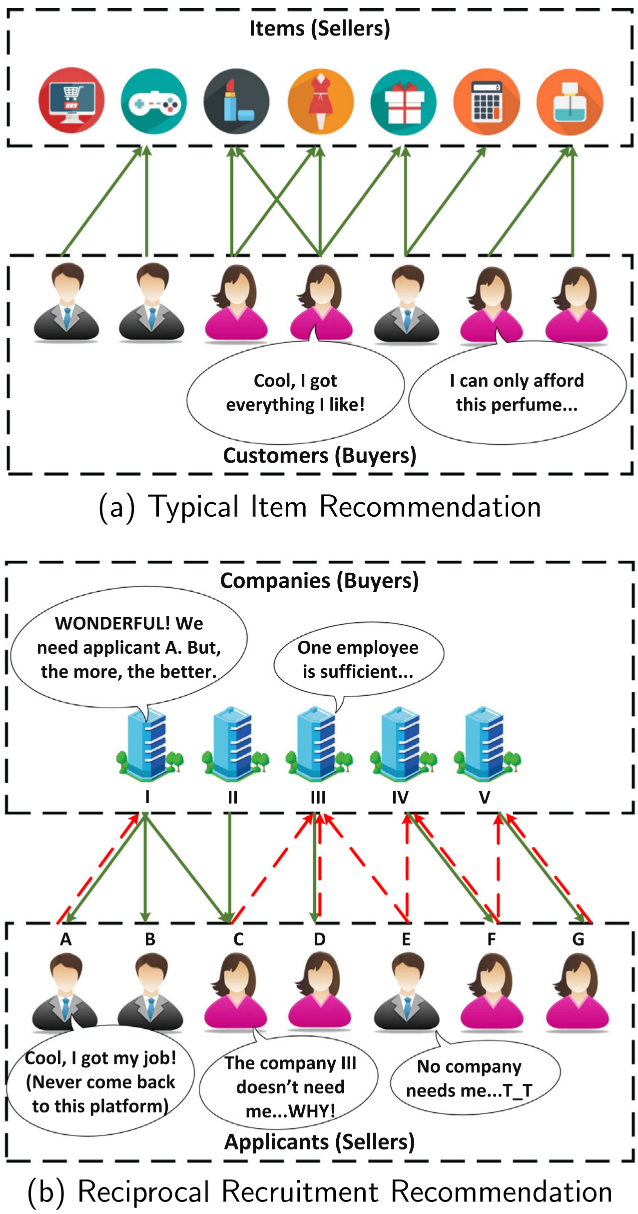

Fig. 1. significant differences between traditional and reciprocal recommender systems.

However, in the online recruiting RS (i.e., the reciprocal RS), it is impossible to satisfy the demands of all applicants and companies simultaneously, because talented applicants are attractive to every company meanwhile the job is unique for each applicant. If the reciprocal RS is optimized only considering the effectiveness of recommendations, the majority of applicants may lose ideal jobs and companies cannot hire enough appropriate employees. Also, in the online dating RS which serves heterosexual users, the ideal pairing situation is that the partners are mutually satisfied with each other and all users (i.e., males and females) meet their lifelong partners. However, if the recommendations only depend on females' or males' preference, the majority of users whose individual conditions are worse may lose chances of dating and leave the platform disappointedly. In these scenarios, due to the incomplete information and discrimination,[1] those people who seem less outstanding (i.e., the users who are hardly recommended to others) can be considered as the members of vulnerable groups. The goal of fairness-aware recommendation is to protect the vulnerable groups from the discrimination and treat all users equally (i.e., users are recommended equally) in the reciprocal RS.

In this paper, we concentrate on the research of fairness-aware recommendations in the reciprocal RS and propose an approach to bridge the gap of techniques between typical and reciprocal recommendations. To overcome the problem of mistreatment, we optimize the reciprocal recommendation approach based on Walrasian equilibrium. Walrasian equilibrium is an economic concept which aims to guarantee the equilibrium between demand and supply while meeting the preference of both parties as far as possible. In other words, each user has the opportunity to be recommended, and the effectiveness of the recommendation will be considered. Based on Walrasian equilibrium, such as the online recruiting RS, the outstanding applicants will still obtain the satisfying job, and the vulnerable applicants may find the appropriate positions in the market. Thus, we cast the task of reciprocal recommendation as Walrasian equilibrium problem and optimize Walrasian equilibrium based on three criteria of fairness between parties (e.g., applicants and companies) in the recommendation: (1) the disparity of service; (2) the similarity of mutual preference; (3) the equilibrium of demand and supply. First, the disparity of service describes the different worths of companies and applicants to the reciprocal RS. For example, the registration of famous companies will attract more applicants. Thus, the reciprocal RS is more concerned about the satisfaction of companies than that of applicants. Second, the similarity of mutual preference describes the similar satisfaction of recommendation between buyers and sellers. In other words, a company and an applicant who are mutually interested in each other comprise the ideal hiring situation. Third, the equilibrium of demand and supply describes the concept of market clear in the economic system. In the online recruiting RS, the market clear means each applicant has an appropriate job and each company hires sufficient employees. According to the equilibrium of demand and supply, the reciprocal RS should optimize the allocation to meet the demand of each party while considering the satisfaction of recommendations. The demand for companies can be understood as the number of employees they plan to hire.

However, in practical scenarios, there exist contradictions between these criteria of fairness and the satisfaction of individuals. For example, if the reciprocal RS extremely concentrates on the satisfaction of individuals (i.e., applicants or companies), the recommendations become more satisfying rather than more suitable to individuals. In other words, major companies are more recommended to applicants rather than small firms because most applicants prefer to major companies which are capable of providing

[1] In fact, the majority of unfair recommendation is caused by the imbalanced data which is a special case of cold-start problem. However, the imbalance data is generated due to the human selection with the unfair discrimination.

stable jobs and opportunities of leaning. Hence, it is nontrivial for start-ups (i.e., small firms), which are considered as vulnerable groups, to attract superior applicants. On the other hand, if the reciprocal RS only considers the fairness of recommendations or the market clearing, individuals would be equally and randomly recommended to others, but the performance of recommendations cannot be guaranteed. From the views of companies and applicants, both of them want to seek for the most satisfying choices ignoring mutual preferences. However, from the view of reciprocal RSs, we are trying to provide suitable recommendations which concentrate on the success rate of recruitment for individuals (i.e., the fairness and satisfaction of recommendations), especially for vulnerable groups. Therefore, to solve such contradictions in the fairness-aware reciprocal recommendation, we employ a multi-objective optimization approach to optimize the satisfaction of individuals, the fairness of recommendations, and the market clearing simultaneously based on the disparity of service, the similarity of mutual preference, and the equilibrium of demand and supply. To the best of our knowledge, this is the first attempt to combine the economic theory (i.e., Walrasian equilibrium) with the fairness-aware recommendation in the reciprocal RS. Our contribution can be:

- We cast the task of fairness-aware recommendations as a Walrasian equilibrium problem, and propose a generalized strategy of fairness-aware recommendations in reciprocal RSs.
- We define the fairness as an optimization considering the disparity of service, the similarity of mutual preference, and the equilibrium of demand and supply simultaneously, and apply the multi-objective optimization to solve this problem effectively.
- We evaluate the proposed algorithm on two real-world databases (i.e., online recruitment and dating) and discuss the effect of fairness on the performance of reciprocal recommendations.

The rest of this paper is organized as follows: Section 2 describes relevant works in the reciprocal recommendation, the multi-objective optimization, and the fairness-aware recommendation; Section 3 presents the details of our fairness-aware recommendation approach in the reciprocal RS; Section 4 discussed and analyzed the experimental results; Finally, Section 5 summarizes our work and introduces the future research in this domain.

2. Related work

In this section, we introduce some significant works in the reciprocal recommendation, the fairness-aware recommendation, and the multi-objective optimization.

2.1. Reciprocal recommendation

The reciprocal RS is a specific type of recommender systems where users can be both subjects and objects. The practical scenarios of reciprocal RS are online dating, recruitment, and mentor–mentee matching. Different from conventional cross-domain recommender systems [7,8], the reciprocal RS concentrates on the mutual recommendation of users who belong to different parties. Pizzato et al. was the first group to define and solve the problem of reciprocal recommendations [9]. Compared with the typical RS, they proposed that there are four major characteristics in the reciprocal RS: (1) a successful recommendation is determined by both parties in the recommendation; (2) a user is not only a subject but also an object, therefore, users are willing to show their profiles for selling themselves. However, according to some constraints of privacy protection, a user should not be recommended to many others; (3) the cold-start is an acute problem because a user may leave the reciprocal RS when she receives a successful recommendation (e.g., online dating and recruitment); (4) if a user abandons the initiative to choose a recommendation, she can also receive a successful transaction because she can be reactive due to the selection from another user. Based on the general definition of reciprocal RS, some research has been proposed to solve the problem of reciprocal recommendations in online dating [1,2,10], recruitment [11,12], and general reciprocal RSs [5,13–15]. Li et al. proposed a representative and generalized framework for reciprocal RSs where they considered the relationships of users as a bipartite graph [5]. The bipartite graph simultaneously considered the local preferences (i.e., the mutual preferences of users) and the global utilities (i.e., the utility of the entire reciprocal RS) to address the specific problem in the reciprocal RS effectively. Ting et al. [15] dived into the research of the potential correlation between the typical and reciprocal RS and proposed a transfer-learning based collaborative filtering (CF) model to improve the performance of typical CF in the reciprocal recommendations. However, the number of studies in reciprocal RSs is still limited, and some significant efforts are expected. In addition, the majority of existing reciprocal RSs focused on improving the performance of recommendations without the consideration of crucial problems (i.e., diversity and fairness of recommendations) in practical scenarios. The ACM RecSys Challenge 2017 was focusing on the cold-start problem of job recommendation [16], and many approaches were proposed to address the practical problem in the social network XING [4,17,18]. However, they concentrated on recommending jobs to users and ignored the satisfaction of companies which published the recruitment.

2.2. Multi-objective optimization

Solving a multi-objective optimization has become a big challenge in decades. The typical scenario of multi-objective optimization is Pareto efficiency where to search for the appropriate Pareto optimal among Pareto frontier. The multi-objective optimization approaches are categorized as follows: classical methods and evolutionary algorithms [19]. Classical methods mainly contain the weighted sum method, ϵ-constraint method, weighted metric method, value function method, and so on. As the common solution of multi-objective optimization, the scalarization method is to formulate and solve a single-objective optimization problem to ensure that Pareto optimal of the single-objective optimization is also the solution of the multi-objective optimization problem. Besides the linear scalarization [20], the achievement scalarizing function [21,22] is also an effective approach to solve the multi-objective optimization problem. When the preference information of the multi-objective optimization is unknown, the global criterion is used to determine the optimization in the scalarization. On the other hand, as the popular solver, evolutionary algorithm is the main strategy for addressing multi-objective optimization effectively and efficiently. Extremal optimization, which is a representative evolutionary algorithm strategy, has become popular in recent years due to its simple implementation and efficiency. Zeng et al. are the first group to extend extremal optimization to the multi-objective problem (MOEO) for designing the optimal fractional order proportional–integral–derivative (FOPID) [23]. In addition, Zeng et al. proposed an improved multi-objective population-based extremal optimization with polynomial mutation (IMOPEO-PLM) and the extensive experimental results show the effectiveness of IMOPEO-PLM [24]. The similar technique is applied in the multi-area interconnected power system [25]. In addition, based on the interactivity of optimization, the multi-objective problem is also categorized into non-interactive and interactive methods where the interactivity determines that whether a decision maker can participate in the procedure of optimization to search for the appropriate Pareto optimal iteratively [26,27]. Also, to solve a multi-objective optimization, observing the visualization of Pareto frontier is also a heuristic and effective method. The idea of visualization is similar to the posterior method which aims to find out all the Pareto optimal solutions and select the most appropriate one [28,29]. In this paper, the parameterized Achievement Scalarizing Function (ASF) is utilized to solve the multi-objective problem [22]. Although ASF is not designed for optimal global solutions, the interactive mechanism of parameterized ASF (i.e., improvement of ASF) makes decision makers participate in the procedure of optimization (i.e., reference point) in reciprocal fairness-aware RSs (detailed in Section 3.5). In other words, due to the complexity of specific reciprocal RSs (i.e., recruitment), decision-makers need to dynamically adjust recommendation lists for balancing the effectiveness and fairness of recommendations.

2.3. Fairness-aware recommendation

After improving the user acceptance, diversity, and the serendipity of recommendations, considering the fairness-aware recommendation has become new research in the recommender system. The earliest research on the fairness in the recommender system is the loan recommendation for micro-finance services where Lee et al. proposed the fairness between lenders and borrowers to maximize the opportunity of successful matching [30]. Recent years, the research of fairness-aware recommendations comprises two areas: (1) the group recommendation [31–35] and (2) the multi-sided recommendation [36–38]. Yao et al. concentrated on biased data which cause unfair recommendation against minority groups and proposed several fairness-aware metrics for any leaning objective [31]. Stratigi et al. proposed a collaborative filtering-based group recommendation in the health domain [32]. In detail, they tried to provide precise and fairness-aware suggestions to a caregiver who takes charge of a group of patients that make each patient be equally treated. Serbos et al. proposed a fairness-aware package-to-group recommendation approach using a greedy strategy to solving the fairness-aware objective which is considered as an NP-hard problem [35]. Modani et al. defined a novel diversity named representative diversity which quantified the interest of users and proposed a multi-sided fairness-aware recommender system using the re-ranking strategy to improving representative diversity [36]. Burke et al. defined a concept of balanced neighborhood and proposed an improved version of the sparse linear method to promoting personalized recommendations while guaranteeing the fairness between users and items in consumer-centered and provider-centered recommender systems, respectively [37,38]. Among the research of fairness-aware group recommendations, as a significant work, Lin et al. defined the fairness as the difference between the maximum and minimum utilities of users in the group, and cast the fairness-aware recommendation as a multi-objective optimization problem simultaneously improving the utility and fairness of the group [33,34]. Also, the research in the fairness-aware reciprocal recommendations are still limited and focus on a specific scenario. More efforts on the generalized fairness-aware reciprocal recommendations are expected to overcome the limitation of current research.

3. Methodology

3.1. Walrasian equilibrium

Walrasian equilibrium [39] (i.e., the competitive equilibrium) is a famous concept in the decentralized economic system, and Walrasian equilibrium is defined based on three parts: (1) maximizing the utility[2] of consumers (i.e., buyers), (2) maximizing the profit of firms (i.e., sellers), and (3) the market clear. The definition of Walrasian equilibrium perfectly describes the essential target of reciprocal RSs where (1) and (2) refer to the effectiveness of recommendation and (3) refers to the fairness of recommendation. In the fairness-aware reciprocal recommendations, the first two parts can be considered as a typical task to maximize the satisfaction of recommendation while considering the fairness to buyers and sellers. However, the primary goal of Walrasian equilibrium is to find the optimal price vector based on the demands to consumers and the quantity of production to firms. In the generalized reciprocal RS, we can relax the condition of Walrasian equilibrium to find the optimal allocation of buyers and sellers instead of the optimal price vector. In other words, we treat each seller equally (i.e., the same price) and regard the demand of buyer as the budget. Therefore, combined with our proposed three criteria of fairness in the generalized reciprocal RS, we redefine Walrasian equilibrium as below:

Definition 1 (*Walrasian Equilibrium*)**.** Given a decentralized economic system which contains a set of buyers, a set of sellers, the demand of buyers, the supply of sellers, the significance of buyers and sellers (i.e., the disparity of services), and the preferences of buyers and sellers, the conditions of Walrasian equilibrium: (1) maximizing the utility of buyers considering the similarity of mutual preferences to sellers; (2) maximizing the utility of sellers considering the similarity of mutual preferences to buyers; (3) balancing the demand of buyers and the supply of sellers.

The primary goal of our proposed approach is to provide reciprocal recommendations while considering the fairness for vulnerable groups effectively. However, different from users in the typical RS, the users can be both subjects and objects in the reciprocal RS (See Fig. 1). Therefore, the evaluations of the effectiveness and fairness are different in the typical and reciprocal RSs. Also, the typical solution of the Walrasian equilibrium problem, which optimizes the three parts in order, will easily generate the local optima. In the following sections, we will define the effectiveness and fairness based on Walrasian equilibrium and introduce the effective solution of Walrasian equilibrium problem.

3.2. Satisfaction of recommendation

In typical RSs, the preference of a user is learned based on her (his) historical data. The potential rates of candidate objects (e.g., items) to the user can be predicted and ranked using machine learning. However, under scenarios of reciprocal recommendations (i.e., online dating and recruitment), users cannot be only a subject but also an object which is recommended to other users. For instance, the online recruiting system recommends candidate companies to applicants while candidate applicants are also recommended to these companies. Thus, to evaluate the performance of reciprocal RSs, we should apply the utility functions to describe the performance of recommendations for applicants and companies respectively. For the ease of understanding, in

[2] The utility describes the total satisfaction of consumers received from consuming a good or service.

the following sections, we name the parties in the reciprocal recommendation as buyers and sellers, respectively.

The utility of individual in the reciprocal RS can be defined as follows. Assume that there exists a set of buyers $\mathbf{U^b} = \{u_1^b, u_2^b, \ldots, u_m^b\}$ and a set of sellers $\mathbf{U^s} = \{u_1^s, u_2^s, \ldots, u_n^s\}$. The relevance of each candidate seller to the buyer is considered as $rel(u_i^b, u_j^s) \in [rel_{min}, rel_{max}], \forall u_i^b \in \mathbf{U^b}, \forall u_j^s \in \mathbf{U^s}$. Similarly, the relevance of each candidate buyer to the seller is considered as $rel(u_i^s, u_j^b)$. Thus, the utility function of the buyer and the seller is:

$$\begin{aligned}
UTL(u_i^b, \mathbf{U^s}) &= \frac{\sum_{u_j^s \in \mathbf{U^s}} rel(u_i^b, u_j^s) \cdot r_{ij}^s}{|\mathbf{U^s}| \times rel_{max}}, \\
s.t. \quad & \mathbf{R_{i\to s}} = \{r_{ij}^s | j = 1, 2, \ldots, |\mathbf{U^s}|\}, \\
& \sum_{r_{ij}^s \in \mathbf{R_{i\to s}}} r_{ij}^s \le l, r_{ij}^s \in \{0, 1\}, \\
UTL(u_j^s, \mathbf{U^b}) &= \frac{\sum_{u_i^b \in \mathbf{U^b}} rel(u_j^s, u_i^b) \cdot r_{ji}^b}{|\mathbf{U^b}| \times rel_{max}}, \\
s.t. \quad & \mathbf{R_{j\to b}} = \{r_{ji}^b | i = 1, 2, \ldots, |\mathbf{U^b}|\}, \\
& \sum_{r_{ji}^b \in \mathbf{R_{j\to b}}} r_{ji}^b \le l, r_{ji}^b \in \{0, 1\},
\end{aligned} \tag{1}$$

where $|\mathbf{U^b}|$ and $|\mathbf{U^s}|$ means the number of users in $\mathbf{U^b}$ and $\mathbf{U^s}$, respectively. l is the predefined maximum length of recommendation list. r_{ij}^s demonstrates that whether the j_{th} seller is recommended to the i_{th} buyer or not. In this paper, $rel(u_i^b, u_j^s)$ is demonstrated using the rating of the corresponding seller to the buyer. $rel(u_j^s, u_i^b)$ is described in the similar way.

The concept of social welfare from the welfare economics [40] is used to define the satisfaction of recommendations for users (i.e., buyers and sellers) in the reciprocal RS. The social welfare is a metric that measures the allocation of resources and goods in the market. Different from typical RSs, the reciprocal RSs should simultaneously consider the social welfare (i.e., the satisfaction of recommendations) for both buyers and sellers. Therefore, based on the definition of social welfare [41] and the utility of individual, the overall satisfaction of recommendations (SR) for buyers and sellers is respectively described as below:

$$\begin{aligned}
SR(\mathbf{G^{U^b}}, \mathbf{U^s}) &= \frac{1}{|\mathbf{G^{U^b}}|} \sum_{g \in \mathbf{G^{U^b}}} \frac{1}{|g|} \sum_{u_i^b \in g} UTL(u_i^b, \mathbf{U^s}), \\
SR(\mathbf{G^{U^s}}, \mathbf{U^b}) &= \frac{1}{|\mathbf{G^{U^s}}|} \sum_{g \in \mathbf{G^{U^s}}} \frac{1}{|g|} \sum_{u_j^s \in g} UTL(u_j^s, \mathbf{U^b}).
\end{aligned} \tag{2}$$

where $\mathbf{G^{U^b}}$ and $\mathbf{G^{U^s}}$ represents the set of buyer and seller groups, respectively. g demonstrates an independent group in $\mathbf{G^{U^b}}$ (or $\mathbf{G^{U^s}}$) where the buyers (or sellers) in g have some similarities. The detail of groups will be mentioned in Section 4. W.l.o.g., we defined the satisfaction of recommendation and following concept under groups to meet the demand of group recommendations.[3] Because the reciprocal RS consists of both buyers and sellers, neither $SR(\mathbf{G^{U^b}}, \mathbf{U^s})$ nor $SR(\mathbf{G^{U^s}}, \mathbf{U^b})$ can demonstrate the overall performance. To address this problem, we jointly consider $SR(\mathbf{G^{U^b}}, \mathbf{U^s})$ and $SR(\mathbf{G^{U^s}}, \mathbf{U^b})$. In addition, according to the asymmetric and incomplete information in practical scenarios, the performance of recommendation for buyers and sellers is different (i.e., the disparity of service). To evaluate the performance of reciprocal RS considering the disparity of service between buyers and sellers, we utilize the following equation to measure the performance of the current reciprocal RS:

$$SR(\mathbf{G}, \mathbf{U}) = \alpha \cdot SR(\mathbf{G^{U^b}}, \mathbf{U^s}) + (1 - \alpha) \cdot SR(\mathbf{G^{U^s}}, \mathbf{U^b}), \tag{3}$$

where $\mathbf{G} = \{\mathbf{G^{U^b}}, \mathbf{G^{U^s}}\}$, $\mathbf{U} = \{\mathbf{U^b}, \mathbf{U^s}\}$, and $\alpha \in (0, 1)$ is a predefined parameter to describe the disparity of service between the buyers and sellers.

3.3. Similarity of mutual preference

In scenarios of reciprocal RS, the preferences of users do not only determine the recommendations for individuals, but also affect the effectiveness of pairing buyers and sellers. For example, in a typical RS, a successful recommendation means that any recommendation in the list meets the user preference. However, in a reciprocal RS such as the online recruiting network, an applicant and a company who are mutually interested in each other comprise an ideal hiring situation (i.e., the successful recommendation). In other words, the recommendations would be successful only if the buyer and the seller have the similar mutual preferences. According to this characteristic of reciprocal RSs, besides considering the intersection of the buyer and seller preference lists as the recommendations, the consistent mutual preference of buyers and sellers is also a significant factor to improve the performance of recommendations. To consider the similarity of mutual preference, for each buyer and seller, we define the consistent mutual preference as follows:

$$\begin{aligned}
MP(u_i^b, \mathbf{U^s}) &= \sum_{u_j^s \in \mathbf{U^s}} |rel(u_i^b, u_j^s) - rel(u_j^s, u_i^b)| \cdot r_{ij}^s, \\
s.t. \quad & \mathbf{R_{i\to s}} = \{r_{ij}^s | j = 1, 2, \ldots, |\mathbf{U^s}|\}, \\
& \sum_{r_{ij}^s \in \mathbf{R_{i\to s}}} r_{ij}^s \le l, r_{ij}^s \in \{0, 1\}, \\
MP(u_j^s, \mathbf{U^b}) &= \sum_{u_i^b \in \mathbf{U^b}} |rel(u_j^s, u_i^b) - rel(u_i^b, u_j^s)| \cdot r_{ji}^b, \\
s.t. \quad & \mathbf{R_{j\to b}} = \{r_{ji}^b | i = 1, 2, \ldots, |\mathbf{U^b}|\}, \\
& \sum_{r_{ji}^b \in \mathbf{R_{j\to b}}} r_{ji}^b \le l, r_{ji}^b \in \{0, 1\},
\end{aligned} \tag{4}$$

Similar to the definition of the satisfaction of recommendations (i.e, Eq. (2)), we define the similarity of mutual preference (SP) as below:

$$\begin{aligned}
SP(\mathbf{G^{U^b}}, \mathbf{U^s}) &= \frac{1}{|\mathbf{G^{U^b}}|} \sum_{g \in \mathbf{G^{U^b}}} \frac{1}{|g|} \sum_{u_i^b \in g} MP(u_i^b, \mathbf{U^s}), \\
SP(\mathbf{G^{U^s}}, \mathbf{U^b}) &= \frac{1}{|\mathbf{G^{U^s}}|} \sum_{g \in \mathbf{G^{U^s}}} \frac{1}{|g|} \sum_{u_j^s \in g} MP(u_j^s, \mathbf{U^b}),
\end{aligned} \tag{5}$$

Thus, the similarity of mutual preference in the current reciprocal RS is defined as below:

$$SP(\mathbf{G}, \mathbf{U}) = \alpha \cdot SP(\mathbf{G^{U^b}}, \mathbf{U^s}) + (1 - \alpha) \cdot SP(\mathbf{G^{U^s}}, \mathbf{U^b}). \tag{6}$$

Note that, our goal is to minimize Eq. (6) to optimize the recommendation list to narrow the gap of mutual preferences between buyers and sellers. However, when the relevance between buyer and seller is small, the gap will also be small. To avoid the problem of low relevances, we adopt δ (i.e., a baseline of relevance) to filter the recommendations that are highly relevant to buyers and sellers.

[3] Our approach is appropriate for both individual and group recommendations. Without loss of generality, we define the reciprocal recommendation as a group recommendation problem which can also describe the task of personalized recommendation.

3.4. Equilibrium of demand and supply

For the typical recommender scenario (e.g., Amazon), optimizing the satisfaction of recommendation and the similarity of mutual preference may boost sales, but the actual quantity of demand, the purchasing power of buyers, and the supply of sellers are often ignored.

The critical characteristic of a decentralized economic system is that buyers are free to choose what they want but given the constraints. On the one hand, the buyer have the affordable consumption bundle, in other words, the consumption must be below the budget. On the other hand, the quantity of items is limited, which means some demands of buyers cannot be satisfied. In addition, some items will be unsalable because they have never been recommended to buyers (i.e., the long-tail problem). This is also a big challenge in the reciprocal RS. To overcome this problem, we should equally recommend each user while considering the budget and maintaining the satisfaction of recommendations and the similarity of mutual preference. Therefore, in the reciprocal the equilibrium of demand and supply (*EDS*) is defined as below:

$$EDS(\mathbf{G^{U^b}}, \mathbf{U^s}) = \frac{1}{|\mathbf{G^{U^b}}|} \sum_{g \in \mathbf{G^{U^b}}} \frac{1}{|g|} \sum_{u_i^b \in g} \left| \frac{rec_{u_i^b} - \beta Req(u_i^b)}{\beta Req(u_i^b)} \right|,$$
$$EDS(\mathbf{G^{U^s}}, \mathbf{U^b}) = \frac{1}{|\mathbf{G^{U^s}}|} \sum_{g \in \mathbf{G^{U^s}}} \frac{1}{|g|} \sum_{u_j^s \in g} \left| \frac{rec_{u_j^s} - \beta Req(u_j^s)}{\beta Req(u_j^s)} \right|, \quad (7)$$

where $rec_{(\cdot)}$ presents the number where a user has been recommended to others, and $Req(\cdot)$ demonstrates the demand of a buyer or a seller. To understand the concept of $Req(\cdot)$, we explain an example of applicants and companies in the reciprocal recruiting RS. Generally, a company can employ many employees, however, an applicant can only have a job. The number of employees that a company hope to employ or the number of jobs that an applicant can have is defined as $Req(\cdot)$. In the ideal situation, we hope that the number of successful recommendations is precisely equal to the demand of the individuals (i.e., buyers and sellers). However, in practical RSs, if the number of recommendations is similar to the actual demand of individuals, the diversity of recommendations cannot be guaranteed. For example, if an applicant can only have a job, based on Walrasian equilibrium, the applicant will be recommended to a specific company. To overcome the limitation, we adopt β to tune the ratio of the recommendations and the demand. Thus, the overall equilibrium of demand and supply in the reciprocal RS can be defined as follows:

$$EDS(\mathbf{G}, \mathbf{U}) = \alpha \cdot EDS(\mathbf{G^{U^b}}, \mathbf{U^s}) + (1-\alpha) \cdot EDS(\mathbf{G^{U^s}}, \mathbf{U^b}). \quad (8)$$

In addition, the goal of our propose approach is to maximize $SR(\mathbf{G}, \mathbf{U})$ and minimize $SP(\mathbf{G}, \mathbf{U})$ and $EDS(\mathbf{G}, \mathbf{U})$ simultaneously. Therefore, we apply $\exp^{-SR(\mathbf{G},\mathbf{U})}$ for the optimization instead of using $SR(\mathbf{G}, \mathbf{U})$ to guarantee the consistent direction of the optimization task.

3.5. Multi-objective optimization

For simultaneously considering the satisfaction of recommendations, the similarity of mutual preference, and the equilibrium of demand and supply, the recommendation task becomes a multi-objective optimization problem. We formulate the fairness-aware reciprocal recommendation problem as below:

Task 1. *Given the set of buyers* $\mathbf{U^b}$*, the set of sellers* $\mathbf{U^s}$*, the goal of the fairness-aware reciprocal recommendation is to recommend the buyers* $\mathbf{U^b}$ *and the sellers* $\mathbf{U^s}$ *to each other while minimizing* $\exp^{-SR(\mathbf{G},\mathbf{U})}$ *(the satisfaction of recommendations, see Eq. (3)),* $SP(\mathbf{G}, \mathbf{U})$ *(the similarity of mutual preference, see Eq. (6)), and* $EDS(\mathbf{G}, \mathbf{U})$ *(the equilibrium of demand and supply, see Eq. (8)) simultaneously.*

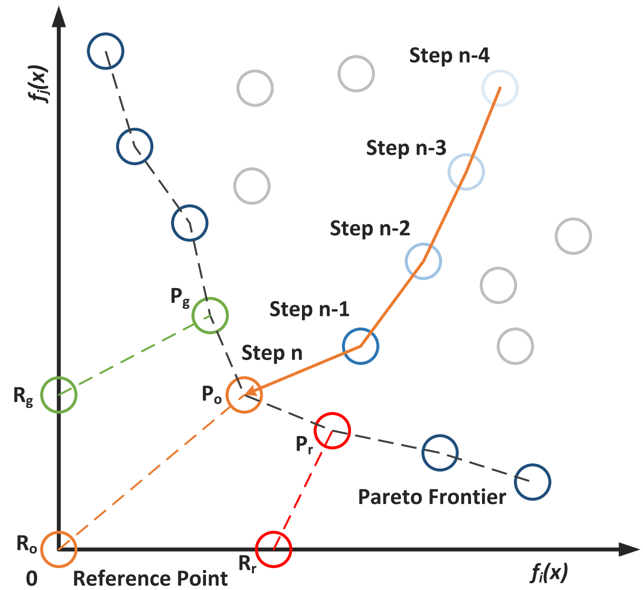

Fig. 2. The procedure to search for the Pareto optimal based on the reference point.

Pareto efficiency, which is an essential concept in economic systems, is generally applied to describe the multi-objective optimization. The definition of Pareto efficiency is:

Definition 2 (*Pareto Efficiency*)**.** Given a multi-objective optimization problem where each objective function is presented as $\{f_i | i = 1, 2, \ldots, m\}$, the values of objective functions are denoted as $S = \{s_1, s_2, \ldots, s_m\}$ which is considered as a solution of the multi-objective optimization problem. A solution $\hat{S}$ dominates S if $\forall i \in [1, m], S_i \leq \hat{S}_i$ and $\exists j \in [1, m], S_j < \hat{S}_j$. If there does not exist other solution dominates $\hat{S}$, the solution $\hat{S}$ is considered as a Pareto optimal. The goal of Pareto efficiency is to find the Pareto frontier which is the set of Pareto optimal.

The typical approach to address multi-objective optimization problems is the scalarization. The core of scalarization is to assign a weight for each objective function and consider the sum of weighted objective functions as a single objective optimization problem. The disadvantage of scalarization is that the target of optimization is obscure. In other words, the Pareto frontier can be available by tuning the weights of objective functions. However, it is hard to search for the best Pareto optimal among the frontier. For example, if we aim to minimize the satisfaction of recommendations while slightly relaxing the similarity of mutual preference, it is non-trivial to quantitatively assign the weights of $\exp^{-SR(\mathbf{G},\mathbf{U})}$ and $SP(\mathbf{G}, \mathbf{U})$. Therefore, it is necessary to overcome the limitation of scalarization to find the appropriate Pareto optimal.

In this paper, we employ the parameterized ASF, which was proposed by Nikulin et al. [22], to solve the multi-objective optimization problem. Besides the assignment of weights to objective functions, we can use a reference point to determine the direction of optimization in the parameterized ASF. Fig. 2 illustrates the procedure to search for the unique Pareto optimal. Each circle on the black dashed line (i.e., Pareto frontier) is a Pareto optimal. R_g, R_o, and R_r are the selected reference points. P_g, P_o, and P_r

can be the solutions (i.e., Pareto optimal) if the reference point is not defined. From Fig. 2, the value of $f_j(x)$ under P_g is larger than that under P_r. In other words, if we aim to find out the most appropriate Pareto optimal, we have to enumerate the set of Pareto optimal and compare with each other. However, if there exists a reference point (i.e., R_g, R_o, or R_r), the reference point can guide the direction of optimization procedure to obtain the corresponding Pareto optimal (i.e., P_g, P_o, and P_r). Therefore, compared with the traditional scalarization, the parameterized ASF is capable of finding out the appropriate Pareto optimal more effectively. The parameterized ASF is defined as below:

$$S^k(f(x),\lambda) = \max_{I^k \subseteq N:|I^k|=k} \left\{ \sum_{i \in I^k} \max\left[\lambda_i (f_i(x) - f_i^R), 0\right] \right\}, \quad s.t. \quad \sum_{i \in N} \lambda_i = 1 \tag{9}$$

where $\lambda = \{\lambda_i | i \in N, \lambda_i > 0\}$ represents the weights of corresponding objective functions, $f(x) = \{f_i(x)|i \in N\}$ represents the set of $|N|$ objective functions and $f^R = \{f_i^R|i \in N\}$ represents the reference point. Thus, solving a multi-objective optimization problem is equal to:

$$\min_{x \in X} S^k(f(x),\lambda), \tag{10}$$

where k is an important parameter to the parameterized ASF. When $k = 1$, $S^1(f(x),\lambda)$ is equal to the largest value among the objective functions. When $k = n$, $S^n(f(x),\lambda)$ is equal to the sum of objective functions. Therefore, tuning the parameter k will change the solution of the optimization problem. The parameterized ASF can be applied for a specific value k (e.g., $S^1(f(x),\lambda)$ or $S^n(f(x),\lambda)$) or simultaneously for all $k \in N$. In practice, the common utilization of parameterized ASF is to calculate $S^k(f(x),\lambda)$ under all $k \in N$ for reaching the global optima [22]. The optimality guarantee of parameterized ASF has been proved in [42].

Simultaneously minimizing Eqs. (3), (6), and (8) has been proved as a NP-hard problem [33]. In addition, the actual representation of recommendation list is a multi-hot vector of candidates (i.e., subjects or objects) in reciprocal RSs where the value of corresponding position demonstrates that whether the candidate is recommended or not. However, it is necessary to utilize the probability of recommending corresponding candidate as a reference instead of applying the multi-hot vector of candidates as the representation of recommendation lists. Thus, in this paper, the mixed-integer nonlinear programming which is capable of dealing with these problem is applied to solve Eq. (10) [43]. The fairness-aware reciprocal recommendation optimization problem is presented as follows:

$$\begin{aligned} \min S^k(f(x),\lambda) = \min & \max_{I^k \subseteq N:|I^k|=k} \left\{ \sum_{i \in I^k} \max\left[\lambda_i (f_i(x) - f_i^R), 0\right] \right\}, \\ s.t. \quad & f(x) = \{\exp^{-SR(\mathbf{G},\mathbf{U})}, SP(\mathbf{G},\mathbf{U}), EDS(\mathbf{G},\mathbf{U})\}, \\ & \mathbf{R_{b\to s}} = \{\mathbf{R_{i\to s}} | i = 1, 2, \ldots, |\mathbf{U^b}|\}, \\ & \mathbf{R_{s\to b}} = \{\mathbf{R_{j\to b}} | j = 1, 2, \ldots, |\mathbf{U^s}|\}, \\ & \mathbf{R} = \{\mathbf{R_{b\to s}}, \mathbf{R_{s\to b}}\}, \\ & \sum_{i \in N} \lambda_i = 1. \end{aligned} \tag{11}$$

When $k = \{1, 2, 3\}$, the complete forms of objective function are defined as:

$$\begin{aligned} S^1(f(x),\lambda) = \max\{ & \max[\lambda_{SR}(exp^{-SR} - f_{SR}^R), 0], \\ & \max[\lambda_{SP}(SP - f_{SP}^R), 0], \\ & \max[\lambda_{EDS}(EDS - f_{EDS}^R), 0]\}, \\ S^2(f(x),\lambda) = \max\{ & \max[\lambda_{SR}(exp^{-SR} - f_{SR}^R), 0] \\ & + \max[\lambda_{SP}(SP - f_{SP}^R), 0], \\ & \max[\lambda_{SP}(SP - f_{SP}^R), 0] \\ & + \max[\lambda_{EDS}(EDS - f_{EDS}^R), 0], \\ & \max[\lambda_{EDS}(EDS - f_{EDS}^R), 0] \\ & + \max[\lambda_{SP}(SP - f_{SP}^R), 0]\}, \\ S^3(f(x),\lambda) = \; & \max[\lambda_{SR}(exp^{-SR} - f_{SR}^R), 0] \\ & + \max[\lambda_{SP}(SP - f_{SP}^R), 0] \\ & + \max[\lambda_{EDS}(EDS - f_{EDS}^R), 0]. \end{aligned}$$

In this paper, considering the efficiency of model [22], we set $k = 1$ and the overall objective function is defined as below:

$$\min S^1(f(x),\lambda). \tag{12}$$

The detail of parameterized ASF for the fairness-aware reciprocal recommendation is presented in Algorithm 1.

Algorithm 1 Parameterized Achievement Scalarizing Function for Fairness-aware Reciprocal Recommendation

Input:
$\mathbf{R}$: $\{\mathbf{R_s}, \mathbf{R_b}\}$ the rank of objects for each group;
δ: the threshold to guarantee the high relevance of each recommendation to the buyers or sellers;
α: the predefined parameter to describe the disparity of service between the buyers and sellers.;
β: the parameter to control the ratio of the recommendations and the demand;
λ: the parameter to control the importance of objective functions;
$f(x)$: the set of optimized term;
f^R: the reference point of objective functions;
l: the length of recommendation list;
k: the parameter to determine the solution of $S^k(f(x),\lambda)$;
N: the number of maximum iteration.
Output:
$\mathbf{L}$: the recommendation list.

```
Initialize R^0 using the uniform distribution;
Predefine δ, α, β, λ, l;
Set f(x) = {exp^{-SR(G,U)}, SP(G,U), EDS(G,U)};
Set f^R = {0, 0, 0}, k = 1;
Set i = 0;
repeat
    min S^1(f(x), λ);
    Update R^i using sequential quadratic programming;
    i+ = 1;
until S^1(f(x), λ) = 0 or i = N
L ← R^i
return the recommendation list L.
```

4. Experiments

In this section, we conduct the extensive experiments on two real-world datasets to evaluate the fairness-aware recommendation in the reciprocal RS, and mainly discuss the following issues:

- *Parameter*: The performance of WE-Rec for buyers and sellers under different parameters (i.e., disparity of services and top-N recommendations).
- *Loss Function*: The relationship between the structure of our loss function and the performance of WE-Rec.
- *Performance*: The satisfaction of active and inactive users under different parties (i.e., buyers and sellers) using the fairness-aware and fairness-free recommendations.

4.1. Experimental setups

The experiments are conducted on two real-world datasets which comprise a recruiting dataset (i.e., WUZZUF job posts) and a dating dataset (i.e., Speed Dating Experiment). WUZZUF is an online recruiting social network which was established in 2009. The recruiting dataset, which is from the kaggle competition,[4] consists of the details of job posts from companies and applications from applicants on the website of WUZZUF from 2014 to 2016. The recruiting dataset contains 1,954,190 applications from 314,460 applicants including 19,208 jobs. The details of job posts and applications comprise some significant information such as the location of jobs, the category of jobs, and the quantity demand for jobs. These detailed information are used to group companies and applicants. Speed Dating Experiment (SDExp[5]) is a dating dataset which was gathered from participants in experimental speed dating events from 2002–2004 [44]. The dating dataset contains 2490 speed dating records from 274 females and 277 males where each person has a complete profile (e.g., career and habit). The similarity of WUZZUF and SDExp is both of them are reciprocal recommender systems where mutual preferences determine whether the recommendation is successful or not. The essential difference between these two datasets is the quantity of demand for users which is considered as a significant parameter of objective (see Eq. (8)). In SDExp, individuals (i.e., females and males) can have only one partner while companies can have a lot of employees (i.e., applicants) in WUZZUF.

In the experiments, we compare the proposed Walrasian equilibrium-based recommendation (WE-Rec) with several methods (i.e., the fairness-aware and fairness-free recommendations):

- Greedy-LM [33]: the greedy algorithm for the fairness-aware group recommendation based on the Least Misery;
- Greedy-JF [45]: the greedy algorithm for the fairness-aware group recommendation based on the Jain's Fairness;
- IP-Var [33]: the fairness-aware group recommendation based on the Variance using the integer program;
- IP-MMR [33]: the fairness-aware group recommendation based on the Min–Max Ratio using the integer program.
- MSRec [46]: Multi-Stakeholder Recommendation is a multi-dimensional utility framework by utilizing multi-criteria ratings considering the fairness of recommendation.
- RECON [9]: RECON is a reciprocal recommendation algorithm for the online dating where the characteristics of individuals are used to calculate the relevance between parties (i.e., males and females). RECON is a fairness-free recommender system.

In this paper, some baselines focus on the fairness-aware group recommendation while some baselines focus on the personalized recommendation. To establish the same experimental environment, we divide the users into groups for each dataset and tune all baselines into the version of group recommendation. In detail, for the recruiting dataset, we divide users into two parties based on their roles (i.e., applicants and companies). In each party, we further group users based on 14 categories of career. Therefore, there are 14 groups in applicants and 14 groups in companies. Similarly, for the dating dataset, we first divide users into two parties based on their genders (i.e., females and males). In each party, we further group users based on 6 goals of dating. Therefore, there are 6 groups in females and 6 groups in males. In addition, except for RECON, other baselines and WE-Rec are the algorithms to improve the performance of fairness by reranking the recommendation list. Thus, we apply the non-negative matrix factorization (NMF) to predict the relevance between parties in the reciprocal recommendation where sklearn is used to implement NMF and the parameters are set as n_components = 6 and init = 'random' [47]. In addition, some parameters of WE-Rec are predefined based on the experiences. The threshold δ, which guarantees the high relevance of each recommendation to the buyers or sellers, is set as $0.2(rel_{max} - rel_{min})$ in the corresponding datasets. The parameter α is set as 0.6 (companies) and 0.4 (applicants) for WUZZUF and 0.6 (males) and 0.4 (females) for SDExp. The ratio β of the recommendations and the demands is set as 5 (job seekers) and 2 (companies) in the recruiting dataset, and set as 5 (women) and 5 (men) in the dating dataset. The weights of objective functions $\boldsymbol{\lambda}$ are 0.4 (the satisfaction of recommendation), 0.3 (the similarity of mutual preferences), and 0.3 (the equilibrium of demand and supply). The reference point of objective functions f^R is [0 (the satisfaction of recommendation), 0 (the similarity of mutual preferences), 0 (the equilibrium of demand and supply)]. The length of recommendation list l is set as 20. The parameter k to determine the solution of $S^k(f(x), \lambda)$ is set as 1.

To intuitively present the experimental results, we adopted several metrics to evaluate the performance of reciprocal recommendations:

$$Rec@l = \frac{\sum_{u \in \mathbf{B} \cup \mathbf{S}} |R(u) \cap T(u)|}{\sum_{u \in \mathbf{B} \cup \mathbf{S}} |T(u)|}, \quad DCG@l = \sum_{i=1}^{l} \frac{2^{rel_i} - 1}{\log_2(i+1)},$$

$$Prec@l = \frac{\sum_{u \in \mathbf{B} \cup \mathbf{S}} |R(u) \cap T(u)|}{\sum_{u \in \mathbf{B} \cup \mathbf{S}} |R(u)|}, \quad IDCG@l = \sum_{i=1}^{|REL|} \frac{2^{rel_i} - 1}{\log_2(i+1)},$$

$$F1@l = \frac{2Rec@l \cdot Prec@l}{Rec@l + Prec@l}, \quad NDCG@l = \frac{DCG@l}{IDCG@l},$$

where l is the length of recommendation list. $R(u)$ and $T(u)$ represents the recommendation list and the actual behavior (i.e., selection), respectively. REL is the list of user preference which is ordered by the relevance, and $IDCG$ is the maximum DCG which represents the performance of ranking. In addition, we apply the cross-validation to evaluate the effectiveness of WE-Rec, where the 5 folds are split based on the timestamps.

4.2. Results and discussions

Parameter: First of all, we evaluate the performance of the fairness-aware reciprocal recommendations in the disparity of service. Fig. 3 illustrates the performance of NDCG in the recruiting dataset within different setups of α. When $\alpha = 0$, it means that WE-Rec is optimized entirely according to the satisfaction of applicants. When $\alpha = 1$, the recommendations are reranked based on the satisfaction of companies. As observed in Fig. 3, the primary trend on NDCG of applicants and companies is monotonous, but the trend is consistent with the shift on α. In addition, the overall NDCG reaches the peak value at $\alpha = 0.4$, which means WR-Rec is effective when the weight of companies (i.e., 0.6) is larger than that of applicants (i.e., 0.4). The result demonstrates that the satisfaction of recommendation to

4 https://www.kaggle.com/WUZZUF/wuzzuf-job-posts.
5 https://www.kaggle.com/annavictoria/speed-dating-experiment/home.

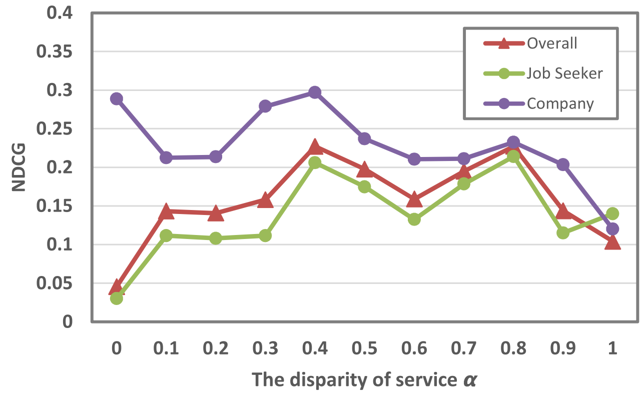

Fig. 3. NDCG of recommendation under α on WUZZUF.

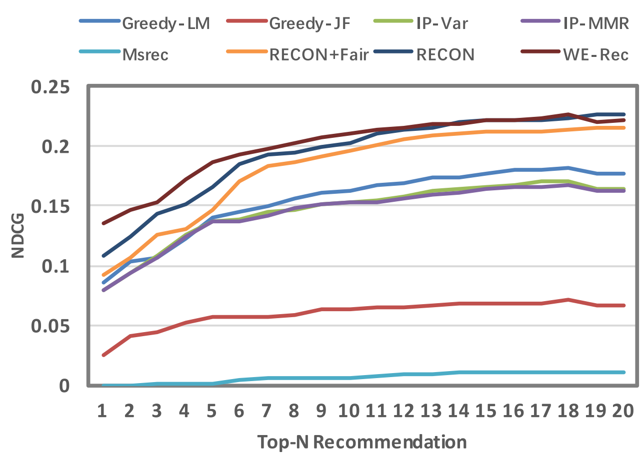

Fig. 4. NDCG of Top-N recommendation on WUZZUF.

companies is more significant than the satisfaction of recommendation to applicants in improving the performance of the current WUZZUF dataset.

In addition, baselines and WE-Rec are evaluated via the top-N recommendations and Fig. 4 illustrates the performance of baselines and WE-Rec on WUZZUF. As observed from Fig. 4, the performance of each method has increased with the increasing number of recommendations and NDCGs have become stationary when N is larger than 15. Therefore, in this paper, a top-20 recommendation is applied to conduct the comparison between baselines and WE-Rec.

Loss Function: The primary goal of WE-Rec is to optimize the satisfaction of individuals, the fairness of recommendations, and the market clearing simultaneously to improve the performance of fairness-aware recommendations. Considering the task of multi-objective optimization, we experiment to investigate the effect of each optimized term to the overall performance. However, WE-Rec provides two ways to adjust the target of optimization: (1) predefine a reference point f^R and (2) predefine β. In this experiment, we modify the reference point f^R to evaluate the significance of each term. Table 2 demonstrates the performance of WE-Rec using different combinations of optimizations where the representations of terms are ① the satisfaction of individuals, ② the fairness of recommendations, and ③ the market clearing. Note that, in both WUZZUF and SDExp, the metric-based performance reaches the top if we only optimized the satisfaction of individuals (i.e., ①). In addition, if we optimized WE-Rec considering other fairness-aware terms (i.e., ② and ③) besides the satisfaction of individuals, the metric-based performance will decrease. This result demonstrates the trade-off between the effectiveness and fairness of recommendations. During the process of optimization, the market clearing is hardly optimized where the convergence of the market clearing happened much early than other terms. It would be a big challenge to optimize the market clearing for improving WE-Rec effectively.

Performance: We compare WE-Rec with other fairness-aware recommendation baselines on the two real-world datasets (the recruiting dataset from WUZZUF and the speed dating dataset from kaggle.com). Except for the fairness-free recommendation RECON, other baselines are fairness-aware recommendations where RECON+Fair is a group recommendation combining the personalized recommendation RECON with the fairness-ware strategy IP+MMR. In addition, RECON+Fair and RECON exploit the match of characteristics between users to define the relevances between users [9], while NMF is used to demonstrate the unobserved relevances for other baselines. Table 3 shows the performance of baselines and WE-Rec in the top-20 recommendation. Besides the overall performance, the performance of different parties is also evaluated on the real-world datasets. The experimental results demonstrate that WE-Rec outperforms other fairness-aware recommendations. Note that, the fairness-aware recommendations (i.e., WE-Rec and RECON+Fair) have the similar performance to the fairness-free recommendation (i.e., RECON), while the fairness-free recommendation (i.e., RECON) outperform any other fairness-aware recommendations. Generally, there exists a trade-off between the effectiveness and fairness of recommendation such as the experimental result in SDExp. However, the fairness-aware recommendation beats the fairness-free recommendation in WUZZUF that also shows the specific characteristic of the online recruiting network. The experimental result of WUZZUF demonstrates that companies (or applicants) prefer the appropriate employees (or jobs) to the top ones. Also, the fairness-free recommendation (e.g., RECON) has a big gap in the performance of recommendation between parties while some fairness-aware recommendations (e.g., WE-Rec and RECON+Fair) narrow down the gap effectively.

Recently, the unfair results are generated due to the imbalanced datasets in some research [48–50]. In recommender systems, the imbalanced dataset is also considered as the problem of a cold start. To further investigate the characteristics of the fairness-aware and fairness-free recommendations, we evaluate the performance of WE-Rec and other baselines considering active and inactive users. We rank all users based on the relevant records within their parties, and top 50% of users are defined as the active users while the remaining users are regarded as inactive users. Tables 4 and 5 shows the performance of WE-Rec and other baselines on WUZZUF and SDExp, respectively. Similar to the experimental results in Table 3, WE-Rec has the same characteristics in the recommendations for the active users of WUZZUF and SDExp. However, the performance of WE-Rec is unsatisfactory for inactive users, especially in SDExp. The major reason is that WE-Rec is concentrating on the balance between parties. Therefore, fairness within parties is hardly considered during the optimization process. Other fairness-aware recommendations also are unsatisfactory in SDExp except for the fairness-free RECON which exploits the match of characteristics between users to define the relevances. However, RECON+Fair, which utilizes the same defined relevances, also has the unsatisfactory results. In other words, the bad performance of fairness-aware recommendations is irrelevant to the definition of relevances between users. Therefore, it is still a challenge to simultaneously the fairness between parties and within parties in reciprocal recommendations.

In addition, we compare WE-Rec to other baselines on runtime to evaluate the efficiency. Table 6 shows the runtime of WE-Rec and baselines in second on SDExp. Due to the different

Table 2
The performance of top-20 recommendation under different loss functions.

Loss Function	Dataset:WUZZUF				Dataset:SDExp			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
①	**0.512**	**0.039**	**0.073**	**0.297**	**0.254**	**0.027**	**0.046**	**0.143**
②	0.199	0.015	0.023	0.078	0.048	0.004	0.007	0.033
③	0.216	0.015	0.024	0.092	0.104	0.010	0.017	0.058
①+②	0.468	0.031	0.053	0.226	0.184	0.024	0.041	0.112
①+③	0.493	0.037	0.062	0.275	0.232	0.026	0.044	0.136
②+③	0.216	0.015	0.024	0.092	0.048	0.004	0.007	0.033
①+②+③	0.453	0.030	0.051	0.221	0.174	0.024	0.040	0.102

Table 3
The performance of top-20 recommendation in the reciprocal recommender system.

Algorithm	Dataset: WUZZUF											
	Overall				Applicants				Companies			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	0.338	0.021	0.038	0.177	0.360	0.018	0.034	0.183	0.263	0.031	0.049	0.159
Greedy+JF	0.126	0.007	0.013	0.066	0.140	0.007	0.013	0.064	0.079	0.006	0.011	0.074
IP+Var	0.298	0.016	0.030	0.164	0.336	0.017	0.032	0.180	0.165	0.015	0.025	0.109
IP+MMR	0.298	0.016	0.030	0.163	0.336	0.017	0.032	0.179	0.165	0.015	0.025	0.107
MSrec	0.065	0.003	0.006	0.019	0.079	0.004	0.008	0.023	0.016	0.001	0.002	0.004
RECON+Fair	**0.434**	**0.027**	**0.048**	0.216	0.383	0.019	0.036	0.190	**0.608**	**0.052**	**0.089**	**0.304**
RECON	0.425	0.026	0.047	**0.227**	**0.393**	**0.020**	**0.037**	**0.206**	**0.537**	**0.048**	**0.081**	**0.297**
WE-Rec	**0.453**	**0.030**	**0.051**	**0.221**	**0.436**	**0.029**	**0.047**	**0.215**	0.477	0.031	0.054	0.247
Algorithm	Dataset: SDExp											
	Overall				Females				Males			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	0.164	0.021	0.035	0.099	0.169	0.021	0.036	0.102	0.161	0.020	0.035	0.099
Greedy+JF	0.106	0.009	0.016	0.096	0.106	0.009	0.015	0.090	0.107	0.009	0.016	0.100
IP+Var	0.107	0.015	0.025	0.060	0.085	0.011	0.019	0.049	0.129	0.018	0.030	0.070
IP+MMR	0.140	0.014	0.025	0.067	0.125	0.014	0.025	0.061	0.155	0.014	0.025	0.072
MSrec	0.114	0.010	0.018	0.088	0.081	0.008	0.013	0.070	0.145	0.013	0.023	**0.105**
RECON+Fair	0.155	0.017	0.029	**0.103**	0.153	0.019	0.032	0.101	0.156	0.015	0.027	0.105
RECON	**0.274**	**0.027**	**0.047**	**0.156**	**0.240**	**0.026**	**0.045**	**0.146**	**0.306**	**0.028**	**0.049**	**0.164**
WE-Rec	**0.174**	**0.024**	**0.040**	0.102	**0.179**	**0.024**	**0.041**	**0.104**	**0.171**	**0.023**	**0.039**	0.099

Table 4
The performance of top-20 recommendation in the active and inactive users of WUZZUF.

Algorithm	Dataset: the active users in WUZZUF											
	Overall				Applicants				Companies			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	0.447	0.030	0.053	0.235	0.467	0.023	0.045	0.234	0.376	0.052	0.080	0.237
Greedy+JF	0.098	0.006	0.011	0.064	0.112	0.006	0.011	0.053	0.050	0.006	0.010	0.101
IP+Var	0.383	0.022	0.040	0.222	0.439	0.022	0.042	0.236	0.191	0.021	0.035	0.173
IP+MMR	0.383	0.022	0.040	0.219	0.439	0.022	0.042	0.233	0.191	0.021	0.035	0.171
MSrec	0.087	0.004	0.008	0.025	0.103	0.005	0.010	0.029	0.032	0.002	0.003	0.008
RECON+Fair	0.479	**0.033**	**0.058**	0.228	0.439	**0.022**	0.042	0.191	**0.614**	**0.069**	**0.112**	**0.356**
RECON	**0.490**	**0.033**	**0.058**	**0.250**	**0.449**	**0.022**	**0.043**	**0.215**	**0.633**	**0.068**	**0.111**	**0.368**
WE-Rec	**0.549**	**0.039**	**0.065**	**0.275**	**0.530**	**0.033**	**0.056**	**0.263**	0.582	0.053	0.086	0.326
Algorithm	Dataset: the inactive users in WUZZUF											
	Overall				Applicants				Companies			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	0.229	0.012	0.023	0.120	0.252	0.013	0.024	0.131	0.151	0.010	0.018	0.081
Greedy+JF	0.155	0.008	0.015	0.068	0.168	0.008	0.016	0.074	0.108	0.006	0.012	0.048
IP+Var	0.213	0.011	0.021	0.106	0.234	0.012	0.022	0.124	0.140	0.008	0.015	0.044
IP+MMR	0.213	0.011	0.021	0.107	0.234	0.012	0.022	0.125	0.140	0.008	0.015	0.044
MSrec	0.043	0.002	0.004	0.014	0.056	0.003	0.005	0.017	0.000	0.000	0.000	0.000
RECON+Fair	**0.389**	**0.021**	**0.039**	**0.203**	0.327	0.016	0.031	**0.189**	**0.602**	**0.035**	**0.066**	**0.252**
RECON	0.357	**0.022**	**0.037**	0.167	**0.343**	**0.024**	**0.038**	0.168	0.372	0.010	0.021	0.169
WE-Rec	**0.360**	0.019	0.036	**0.204**	**0.336**	**0.017**	**0.032**	**0.197**	**0.441**	**0.027**	**0.051**	**0.226**

environments of reproduction, the comparison is divided into two parts: Python and Matlab. As observed from Table 6, WE-Rec is outperformed by other baselines. The experimental result is not surprising because WE-Rec has three objectives, while other baselines only have either one or two objectives.

From the overall experimental results, there still exist several issues should be investigated further. In this paper, the parameter $k = 1$ is utilized to determine the formulation of Eq. (11). In other words, the multi-objective optimization problem is updated according to the maximum loss among the values of objective functions. Therefore, the current solution is the local optima probably. The global optima may be available if we take all possible values of k (i.e., $\{1, 2, 3\}$ in our problem) into account, although the computational cost becomes expensive. In addition,

Table 5
The performance of top-20 recommendation in the active and inactive users of SDExp.

Algorithm	Dataset: the active users in SDExp											
	Overall				Applicants				Companies			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	**0.315**	**0.043**	**0.072**	**0.192**	**0.311**	**0.043**	**0.072**	**0.191**	**0.318**	**0.042**	**0.073**	**0.194**
Greedy+JF	0.111	0.010	0.018	0.156	0.086	0.009	0.015	0.131	0.134	0.012	0.022	0.179
IP+Var	0.208	0.029	0.049	0.118	0.170	0.023	0.038	0.098	0.244	0.035	0.058	0.137
IP+MMR	0.201	0.022	0.039	0.102	0.192	0.023	0.039	0.098	0.210	0.021	0.038	0.105
MSrec	0.115	0.012	0.022	0.095	0.058	0.006	0.011	0.066	0.167	0.018	0.032	0.122
RECON+Fair	0.100	0.014	0.023	0.098	0.147	0.019	0.033	0.131	0.056	0.009	0.014	0.067
RECON	0.272	0.034	0.057	0.151	0.256	0.034	0.057	0.165	0.286	0.034	0.058	0.137
WE-Rec	**0.325**	**0.046**	**0.077**	**0.194**	**0.321**	**0.046**	**0.077**	**0.193**	**0.328**	**0.045**	**0.077**	**0.194**
Algorithm	Dataset: the inactive users in SDExp											
	Overall				Applicants				Companies			
	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG	Rec	Prec	F1	NDCG
Greedy+LM	0.014	0.002	0.003	0.007	0.026	0.003	0.005	0.014	0.013	0.001	0.001	0.003
Greedy+JF	0.102	0.007	0.013	0.035	**0.126**	**0.009**	**0.016**	0.050	0.080	0.006	0.011	0.022
IP+Var	0.007	0.000	0.001	0.002	0.000	0.000	0.000	0.000	0.013	0.001	0.001	0.003
IP+MMR	0.079	0.006	0.011	0.032	0.057	0.006	0.010	0.024	0.100	0.006	0.011	0.039
MSrec	**0.114**	**0.008**	**0.015**	**0.082**	0.105	**0.009**	**0.016**	**0.075**	0.122	0.008	0.015	0.089
RECON+Fair	0.103	0.007	0.013	0.077	0.071	0.006	0.010	0.059	**0.133**	**0.009**	**0.016**	**0.095**
RECON	**0.277**	**0.020**	**0.037**	**0.161**	**0.224**	**0.019**	**0.034**	**0.128**	**0.327**	**0.022**	**0.041**	**0.192**
WE-Rec	0.024	0.002	0.003	0.009	0.036	0.003	0.005	0.016	0.013	0.001	0.001	0.003

Table 6
The comparison of runtime to baselines and WE-Rec on SDExp.

Method	Average Runtime(s)	Environment
RECON	0.106	Python
Msrec	0.426	
Greedy-LM	1.677	Matlab
Greedy-JF	1.605	
IP-Var	97.099	
IP-MMR	71.146	
RECON+Fair	57.398	
WE-Rec	121.427	

The results show that the integer programming is more effective to solve the NP-hard problem than the greedy algorithm in this paper. However, the rich computational complexity and cost limit the practical use of parameterized ASF in the scenarios which demand the efficiency. The greedy algorithm is often utilized to solve this sort of NP-hard problem in an efficient way, but it usually produces the local optimal solutions. To the practical purpose, the greedy algorithm should be tuned to optimize the satisfaction of recommendations, the similarity of mutual preferences, and the equilibrium of demand and supply one by one while maintaining the overall performance of reciprocal RSs. In addition, WE-Rec is an algorithm to rerank the recommendation list by optimizing the satisfaction of recommendations, the similarity of mutual preference, and the equilibrium of demand and supply. In other words, if we apply the Bayesian personalized ranking to predict the relevance between parties instead of using the non-negative matrix factorization, the performance of the reciprocal RS may be different. Therefore, the selection of prediction algorithm is important. In fact, it is a big challenge to solve the multi-objective optimization problem based on an extremely sparse relevance matrix. Finally, the reference point in our experiments is set as a zero vector. The performance of a reciprocal RS should be further improved if we set an appropriate target for each objective function.

5. Conclusion

In this paper, we dive into the research of fairness-aware recommendations in the reciprocal RS and propose an algorithm to rerank the recommendation list considering three criteria based on Walrasian equilibrium: (1) the disparity of service; (2) the similarity of mutual preference; (3) the equilibrium of demand and supply. We conduct the experiments on two real-world datasets (i.e., the recruiting dataset from WUZZUF and the speed dating dataset from kaggle.com) to evaluate WE-Rec. The experimental results demonstrate that considering the fairness between parties can improve the performance of reciprocal recommendations and address the problem of the imbalance between demand and supply. In addition, combining with the consideration of the fairness in each party, the performance of our proposed fairness-aware reciprocal recommendations can be improved further.

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